X. *Observations on Atmospherical Refraction as it affects astronomical Observations; in a Letter from* S. Groombridge, *Esq. to the Rev.* Nevil Maskelyne, *D. D. F. R. S. Astronomer Royal. Communicated by the Astronomer Royal.*

Read March 28th, 1810.

Dear Sir, *Blackheath, 22nd January,* 1810.

The great advantages we derive from the labours and writings of those learned men, who have preceded us; the profound researches in physical astronomy by the mathematicians of the last century; and the improvements in the construction of instruments, effected by the ingenuity of our modern artists: these circumstances should stimulate all those who have inclination and opportunity, to the endeavour of contributing to the common stock of knowledge. I beg leave to add my tribute of admiration to the general voice, for the large share you, Sir, have taken in the promotion of science, not only by your learned publications, but also by the liberal encouragement and assistance you have given to those who have been honoured with your acquaintance. I am, therefore, induced to commit to your protection the inclosed paper, acknowledging myself indebted to your friendly communications for much useful information. I could have wished the task had been performed at an earlier period; but as you are well acquainted with the labour required in the reduction of so many observations by one person, it will appear to have been impracticable. I remain,

Dear Sir,

Your obliged and very obedient servant,

STEPHEN GROOMBRIDGE.

To the Rev. Dr. Maskelyne,
Astronomer Royal, &c. &c.

WHEN I had fixed my four feet transit circle, made by TROUGHTON, it was my intention to make observations of a sufficient number of circumpolar stars, at different altitudes, to ascertain the latitude of my observatory with the greater precision; and also to endeavour to settle the true quantity of refraction: especially as my instrument was better adapted for that purpose, both from the construction and convenient size thereof, than those which had been heretofore made. Being fixed on stone piers, it is not so liable to partial expansion, as those instruments which are supported on brass frames: and having both sides divided, with two microscopes to each face; the same observation has the advantage of four made with a quadrant. I have also contrived, by means of sliding shutters in the roof, to prevent the rays of the sun from falling on any part, excepting on the object glass. By these means I flatter myself that, the observations having been made with great care and attention, the result will prove both satisfactory and useful.

Having proposed to myself the above course of observations, I selected the fifty stars contained in the first table for that purpose, the observations of which, exceed in number

1000. On the quantity of refraction to be assumed at 45°, astronomers not having been agreed, and Dr. MASKELYNE having suggested in the precepts to his folio tables, that Dr. BRADLEY having supposed the sun's parallax $10\frac{1}{3}''$, from which he inferred the refraction at 45° to be 57″; if he had used the true parallax $8\frac{3}{4}''$, he would have found the refraction at the altitude of 45° to be $56\frac{1}{2}''$. I therefore assumed, and have constantly applied that quantity, correcting the same for the barometer and thermometer: the whole by the formula of Dr. BRADLEY.

In the above table, the third column is the mean of the observed zenith distance corrected by the equations to 1st Jan. 1807; the fourth is the mean of the computed refraction, for each observation; the sixth is the sum of the third and fourth colums, which gives the mean double zenith distance of the Pole, according to the assumed refraction in the seventh column; the eighth is the correction of the assumed refraction by the factor found ,02845, which is applied to the seventh column, and gives the true double zenith distance of the Pole in the ninth, the half of which is the corrected co-latitude in the tenth column. It appears that so far as η Ursæ majoris, the sum or difference of the zenith distances in the seventh column are sufficiently uniform to be used for the correction of the refraction. I then proceed to compare the first thirteen stars, where the zenith distance below the Pole is less than 60°, with the twenty-one following. From the former thirteen, the mean of the seventh column is 77° 3′ 53″, 0908; and the mean sum of the refractions in the fifth column is 94,″9377; from the latter twenty-one the means of the same columns are 77° 3′ 50,″5248 and 185,″1357. The difference

TABLE I. *Circumpolar Stars for the Refraction, reduce*

Star.	No Observ.	Corrected Zenith distance. ° ′ ″	Comp. Refraction. ′ ″	Sum or difference. ′ ″	Mean Zenith distance. ° ′ ″	Sum or difference. ° ′ ″	Cor. of Refraction. ″	Sum or diff. of true Zen dis. ° ′ ″	Co-Latitude. ° ′ ″
Ursæ min. Bode 4.	6 7	37 35 10,59 39 27 12,45	0 44,66 0 44,97	1 29,63	37 35 55,25 39 27 57,42	77 3 52,67	2.55	77 3 55,22	38 31 57,
Cephei. Hev. 24.	16 8	37 14 4,10 39 48 16,32	0 42,93 0 48,95	1 31,88	37 14 47,03 39 49 5,27	77 3 52,30	2,61	77 3 54,91	38 31 57,
Polaris.	35 41	36 47 55,60 40 14 28,52	0 42,49 0 47,18	1 29,67	36 48 38,09 40 15 15,70	77 3 53,79	2,56	77 3 56,35	38 31 58,
24 Ursæ minor.	20 7	35 27 34,06 41 34 48,52	0 39,19 0 51,56	1 30,75	35 28 13,25 41 35 40,08	77 3 53,33	2,58	77 3 55,91	38 31 57,
δ Ursæ minor.	27 5	35 5 21,71 41 57 0,22	0 38,78 0 52,42	1 31,20	35 6 0,49 41 57 52,64	77 3 53,13	2,59	77 3 55,72	38 31 57,
Cameiop. Hev. 29.	6 9	33 44 25,40 43 17 56,38	0 38,12 0 53,61	1 31,73	33 45 3,52 43 18 49,99	77 3 53,51	2,61	77 3 56,12	38 31 58,
ω Cephei.	5 7	33 44 11,56 43 18 11,70	0 38,33 0 52,30	1 30,63	33 44 49,89 43 19 4,00	77 3 53,89	2,57	77 3 56,46	38 31 58,
Camelop. Hev. 30.	7 9	32 3 5,85 44 59 13,51	0 36,26 0 57,01	1 33,27	32 3 42,11 45 0 10,52	77 3 52,63	2,65	77 3 55,28	38 31 57,
ε Ursæ minor.	14 10	30 51 25,00 46 10 54.73	0 33,00 1 0,48	1 33,48	30 51 58,00 46 11 55,21	77 3 53,21	2,67	77 3 55,88	38 31 57,
5 Ursæ minor.	8 5	25 4 45,80 51 57 27,75	0 26,00 1 13,80	1 39,80	25 5 11,80 51 58 41,55	77 3 53,35	2,84	77 3 56,19	38 31 58,
β Ursæ minor.	14 8	23 28 12,07 53 34 0,30	0 23,94 1 17,03	1 40,97	23 28 36,01 53 35 17,33	77 3 53.34	2,87	77 3 56,21	38 31 58,
γ 1 Ursæ minor.	5 8	21 3 2.10 55 59 4,63	0 21,35 1 24,18	1 45,53	21 3 23.45 56 0 28.81	77 3 52,26	3,00	77 3 55,26	38 31 57,
γ 2 Ursæ minor.	5 9	21 2 48,68 55 59 18,44	0 21,33 1 24,32	1 45,65	21 3 10.01 56 0 42,76	77 3 52,77	3,01	77 3 55,78	38 31 57,
ο Cephei.	30 5	15 35 5,97 61 26 48,13	0 15,70 1 42,87	1 58,57	15 35 21,67 61 28 31,00	77 3 52,67	3,37	77 3 56,04	38 31 58,
α Dracon.	13 6	13 49 48,80 63 11 53,24	0 13,77 1 55,84	2 9,61	13 50 2.57 63 13 49,08	77 3 51,65	3,69	77 3 55,34	38 31 57,
α Ursæ major.	13 4	11 19 11,17 65 42 25,14	0 11,15 2 4,36	2 15,51	11 19 22,32 65 44 29,50	77 3 51,82	3,86	77 3 55,68	38 31 57,
η Dracon.	26 8	10 29 0,80 66 32 30,11	0 10,27 2 11,18	2 21,45	10 29 11,07 66 34 41,29	77 3 52,36	4,02	77 3 56,38	38 31 58,
ϑ Dracon.	33 6	7 36 53,00 69 24 21,09	0 7,38 2 29,85	2 37,23	7 37 0,38 69 26 50,94	77 3 51,32	4,48	77 3 55,80	38 31 57,
2 Lyncis.	9 7	7 35 31,82 69 25 45,39	0 7,84 2 25,31	2 33,15	7 35 39,66 69 28 10,70	77 3 50,36	4,36	77 3 54,72	38 31 57,
δ Ursæ major.	14 5	6 38 12,07 70 22 53,50	0 6,56 2 39,16	2 45,72	6 38 18,63 70 25 32,66	77 3 51,29	4,71	77 3 56,00	38 31 58,
β Cassiop.	22 9	6 36 56,92 70 24 13,02	0 6,62 2 35,33	2 41,95	6 37 3,54 70 26 48.35	77 3 51,89	4,61	77 3 56,50	38 31 58,
β Ursæ major.	29 6	5 56 42,74 71 4 18,90	0 5,84 2 43,84	2 49,68	5 56 48,58 71 7 2,74	77 3 51,32	4,83	77 3 56,15	38 31 58,
ε Ursæ major.	26 7	5 32 27,53 71 28 28,95	0 5,42 2 49,39	2 54,81	5 32 32,95 71 31 18,34	77 3 51,29	4,97	77 3 56,26	38 31 58,
ζ Ursæ major.	31 7	4 28 5,41 72 32 37,94	0 4,35 3 2,96	3 7,31	4 28 9,76 72 35 40,90	77 3 50,66	5,33	77 3 55,99	38 31 57,
η Persei.	10 9	3 36 58,63 73 23 44,29	0 3,65 3 4,10	3 7,75	3 37 2,28 73 26 48,39	77 3 50,67	5,34	77 3 56,01	38 31 58,

reduced to the mean Zenith Distance on the First of January, 1807. [*To face*

Co-Latitude.	Star.	No. Observ.	* Corrected Zenith distance.	Comp. Refraction.	Sum or difference.	Mean Zenith distance.	Sum or difference.	Cor. of Refraction.	Sum or diff. of true Zen. dis.	La
′ ″			° ′ ″	′ ″	′ ″	° ′ ″	° ′ ″	″	° ′ ″	°
31 57,61	γ Ursæ major.	20 9	3 17 59,02 73 42 35,84	0 3,23 3 12,08	3 15,31	3 18 2,25 73 45 47,92	77 3 50,17	5,56	77 3 55,73	38 3
31 57,45	δ Aurigæ.	9 6	2 47 3,31 74 13 31,93	0 2,84 3 10,28	3 13,12	2 47 6,15 74 16 42,21	77 3 48,36	5,49	77 3 53,85	38 3
31 58,17	ϑ Cassiop.	10 7	2 39 6,33 74 21 22,98	0 2,69 3 16,87	3 19,56	2 39 9,02 74 24 39,85	77 3 48,87	5,68	77 3 54,55	38 3
31 57,95	ϑ Bootis.	30 4	1 16 47,07 75 43 16,44	0 1,24 3 45,70	3 46,94	1 16 48,31 75 47 2,14	77 3 50,45	6,46	77 3 56,91	38 3
31 57,86	γ Persei.	7 9	1 16 20,81 75 43 51,84	0 1,25 3 35,73	3 36,98	1 16 22,06 75 47 27,57	77 3 49,63	6,17	77 3 55,80	38 3
31 58,06	ϑ Ursæ major.	14 9	1 4 52,51 75 55 13,18	0 1,08 3 42,08	3 43,16	1 4 53,59 75 58 55,26	77 3 48,85	6,35	77 3 55,20	38 3
31 58,23	β Dracon.	33 7	0 58 54,36 76 1 2,45	0 0,96 3 51,31	3 52,27	0 58 55,32 76 4 53,76	77 3 49,08	6,61	77 3 55,69	38 3
31 57,64	γ Dracon.	58 8	0 2 57,37 76 56 44,22	0 0,05 4 7,90	4 7,95	0 2 57,42 77 0 52,12	77 3 49,54	7,05	77 3 56,59	38 3
31 57,94	η Ursæ major.	57 10	1 11 11,67 78 10 30,62	0 1,16 4 30,98	4 29,82	1 11 12,83 78 15 1,60	77 3 48,77	7,94	77 3 56,71	38 3
31 58,09	α Persei.	20 12	2 18 14,89 79 17 19,00	0 2,30 4 46,90	4 44,60	2 18 17,19 79 22 5,90	77 3 48,71	8,09	77 3 56,80	38 3
31 58,10	ϑ Persei.	15 8	3 3 48,89 80 2 32,59	0 3,11 5 7,69	5 4,58	3 3 52,00 80 7 40,28	77 3 48,28	8,67	77 3 56,95	38 3
31 57,63	τ Herculis.	5 5	4 41 14,02 81 38 50,33	0 4,47 6 15,64	6 11,17	4 41 18,49 81 45 5,97	77 3 47,48	10,56	77 3 58,04	38 3
31 57,89	Capella.	24 19	5 40 45,74 82 37 51,21	0 5,63 6 47,87	6 42,24	5 40 51,37 82 44 39,08	77 3 47,71	11,44	77 3 59,15	38 3
31 58,02	ψ Ursæ major.	14 5	5 55 19,56 82 52 3,33	0 5,85 7 9,74	7 3,89	5 55 25,41 82 59 13,07	77 3 47,66	12,06	77 3 59,72	38 3
31 57,67	β Aurigæ.	9 11	6 33 13,88 83 29 32,80	0 6,50 7 34,35	7 27,85	6 33 20,38 83 37 7,15	77 3 46,77	12,74	77 3 59,51	38 3
31 57,84	α Cygni.	26 6	6 52 9,82 83 47 39,78	0 6,82 8 23,42	8 16,60	6 52 16,64 83 56 3,20	77 3 46,56	14,13	77 4 0,69	38 3
31 58,19	λ Ursæ major.	9 5	7 35 31,84 84 30 21,78	0 7,62 9 6,55	8 58,93	7 35 39,46 84 39 28,33	77 3 48,87	15,33	77 4 4,20	38 3
31 57,90	ε Aurigæ.	27 8	7 56 31,11 84 51 8,35	0 8,07 9 17,51	9 9,44	7 56 39,18 85 0 25,86	77 3 46,68	15,63	77 4 2,31	38 3
31 57,36	10 Ursæ major.	6 3	8 55 37,75 85 48 26,95	0 9,07 11 17,66	11 8,59	8 55 46,82 85 59 44,61	77 3 57,79	19,02	77 4 16,81	38 3
31 58,00	μ Ursæ major.	14 5	8 59 58,08 85 52 23,49	0 9,01 11 39,45	11 30,44	9 0 7,09 86 4 2,94	77 3 55,85	19,64	77 4 15,49	38 3
31 58,25	γ Androm.	12 4	10 4 1,34 86 54 21,52	0 10,20 13 51,03	13 40,83	10 4 11,54 87 8 12,55	77 4 1,01	23,35	77 4 24,36	38 3
31 58,07	β Bootis.	8 2	10 18 23,26 87 7 35,00	0 10,02 15 0,17	14 50,15	10 18 33,28 87 22 35,17	77 4 1,89	25,33	77 4 27,22	38 3
31 58,13	η Aurigæ.	17 4	10 30 17,75 87 19 18,98	0 10,78 15 3,00	14 52,22	10 30 28,53 87 34 21,98	77 3 53,45	25,38	77 4 18,83	38 3
31 57,99	ζ Aurigæ.	9 2	10 41 4,42 87 29 7,32	0 10,91 15 58,10	15 47,19	10 41 15,33 87 45 5,42	77 3 50,09	26,94	77 4 17,03	38 3
31 58,00	β Persei.	11 6	11 15 42,06 88 1 23,30	0 11,29 18 24,73	18 13,44	11 15 53,35 88 19 48,03	77 3 54,68	31,11	77 4 25,79	38 3

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Co-Latitude.		
°	′	″
38	31	57,86
38	31	56,92
38	31	57,27
38	31	58,45
38	31	57,90
38	31	57,60
38	31	57,84
38	31	58,29
38	31	58,35
38	31	58,40
38	31	58,47
38	31	59,02
38	31	59,57
38	31	59,86
38	31	59,75
38	32	0,34
38	32	2,10
38	32	1,15
38	32	8,40
38	32	7,74
38	32	12,18
38	32	13,61
38	32	9,41
38	32	8,51
38	32	12,89

of the polar distances, divided by the difference of the sum of the refractions, quotes ,0284485; which being increased by unity, is the factor to be multiplied into the assumed refraction, viz. $56\frac{1}{2}''$. Then the mean refraction at 45° will be 56,"5 × 1,02845 = 58,"10734: the co-latitude

$$\frac{77^\circ\ 3'\ 53,''0908 + \overline{94,9377 \times ,02845}}{2} = 38^\circ\ 31'\ 57,''897$$

and the latitude $= 51\ \ 28\ \ 2,103,$

which will be found the same, from the mean of the corrected co-latitudes in the tenth column, as given by the above thirty-four stars.

I have made these deductions from the fixed stars only; and therefore it will be required to compare the result with the sun at the solstices. I have preferred the former, from the greater number of observations to be so obtained, as well as from the more accurate bisection of a star, than the application of a wire to the limb of the sun; I have, however, endeavoured to render the observations of the sun, of equal consequence, by means of two fixed, and one moveable wire; the latter measures a space to one of the fixed wires, distant 29′ 45,"74 from the other, the sum of which is the diameter; the sun passing at the same time through the centre of the field. These wires are applied to the upper and lower limbs of the sun, by direct vision, the eye glass having a vertical, as well as a horizontal motion. The centre of the sun is found, knowing the distance of the extreme fixed wires, from that in the centre of the field; but the refraction must be applied for each limb, the difference of refraction at the winter solstice being 8″; therefore this diminution of the diameter must be noticed. The few opportunities I have had to observe the solstices, have produced only eighteen observations at the summer, and thirteen at the winter, solstice. The proof resulting from this small number is, however, satisfactory; these being reduced to the same period, 1st January, 1807, are as follow:

	App. zen. dis.	Refraction.	Correct.	True zen. dis
sum. sols.	27° 59′ 38,″00	0′ 29,″83	+ 0″85	28° 0′ 8,″68
winter do.	74 52 24,16	3 26,34	5,52	74 55 56,02
				102 56 470
			Latitude	51 28 2,35

The mean obliquity of the ecliptic for the same period will be

74°	55′	56,″02
28	0	8,68
46	55	47,34
23	27	53,67

The new solar tables state the obliquity of the ecliptic for 1800 at 23° 27′ 57″, and taking the decrease at half a second per annum, shews the above result to be very near the truth.

I shall now compare the observations of Lieut. Col. MUDGE, made at the Royal Observatory in 1802, with his zenith sector, to ascertain the difference of latitude between that place and my observatory: the following stars being reduced to the same epoch β γ 45, 46, 51, Draconis; κ, ι, Cygni; γ, η, Ursæ majoris; ι, Herculis, and Capella. The mean difference of the zenith distances of these stars, and my own observations, is 35,″49; which being added to my latitude, shews the latitude of the Royal Observatory to be 51° 28′ 37,″59;

and this subtense of the arc agrees very nearly with that of γ Draconis, by Dr. MASKELYNE, which he has communicated to me, being 35,"31. Several of those made by Lieut. Col. MUDGE were single observations, with the zenith sector in one position only, and therefore were reduced by the mean collimation, found from the other stars; but the result shews the accuracy of his observations. The latitude of Greenwich, when corrected from the mean of circular instruments, as proposed by Mr. POND, and by the refraction for the excess of 57", will not differ more than half a second, from the above determination.

Having found by this hypothesis, the mean astronomical refraction at 45°, I shall proceed to compare the same, with the observations of other astronomers. M. PIAZZI has very ingeniously, and with great labour, made his deduction of the mean refraction, from the observations of Procyon, *a* Lyræ and Aldebaran at equal altitudes, from 38° to 89° 30′ zenith distance, in addition to several circumpolar stars, for his latitude: this advantage having been obtained from his instrument moving in azimuth. The result thereof gives the mean refraction at 45° = 57,"3 into the formula of Dr. BRADLEY, tang. $\overline{z-3r}$; which I have found, as above, 58,"107. M. PIAZZI, therefore, shews in his table of mean refractions, that at $45° = 57,''3 \times \text{tang.}\,\overline{45°-3r} = 57,''2$; which will be = 58",01 in my table. On a comparison with the present French tables, the latter give 58,"2, at 45°, which is in excess of my own. The formula of M. DE LA PLACE, as given in his Mecanique Celeste, which is very elaborate and ingenious, being applied to a constant quantity, 60,"616, as determined from observation, will produce a table of refractions, nearly the same as Dr. BRADLEY'S, though the latter is effected by a more simple formula. M. PIAZZI has shewn from his observations, that the actual refractions, so far as 80° from the zenith, are greater than those assumed by Dr. BRADLEY; and thence to the horizon, the formula of the latter produces an excess, therefore requires a reduction in the quantity. The ninth and tenth columns of the above fifty stars, will show the same result as that of M. PIAZZI; viz. that below 80° the assumed refraction is greater than will be given from observation: and the mean correction required for the four last stars, from 87° to 89°, will be — 0,027, which he determines — 0,029. Those stars from the zenith to 80°, I have already corrected, by assuming a greater constant quantity; whence it follows, that all those equations will be plus, as compared with Dr. BRADLEY; but this will not happen with those at lower altitudes, since his formula thence fails: and it is the correction thereof, which I shall next proceed to investigate.

This problem of the mean refractions, so important to practical astronomy, which has occupied the attention of many mathematicians, has always had in view, to reconcile the known laws of the refraction of a ray of light passing through different media, with the actual quantity deduced from observation. The investigations of various authors are given in the first volume of Mr. VINCE'S Astronomy; and that of Mr. SIMPSON, in his Mathematical Dissertations. It has been proved, that the refractions vary nearly as the tangents of zenith distance, into a constant quantity; and assuming the variation directly as the tangents, the approximation will not be sensibly affected so far as 60°; but comparing these, with those at lower altitudes, as found from observation, it will appear that the refractions vary, as the tangents of zenith distance, minus some multiple of r (the refraction), which call y. Now the value of y will be found by the formula drawn from the investigation given at large in Mr. VINCE's Astronomy, by comparing the observed refraction, at different zenith distances, putting $r =$ refraction, $a =$ zenith distance of the highest star; r' and a' of the lowest star: then

$$y = \frac{\overline{r \times \cot. a} - \overline{r' \times \cot. a'}}{\overline{r'^2 - r^2}}.$$

Applying this formula to the mean refraction of Polaris below the pole $= 49'',105$, compared with the corrected or observed refraction of each of the last three stars; the mean value of $y = 3,3625$. Dr. BRADLEY found by comparing the refraction at 60° with that at 90°, which he supposed 33′ 0″, the value of $y = 2,996$; he therefore assumed 3. M. CASSINI found the same to be 3,226; M. BOUGUER 3,323; and Mr. SIMPSON 2,75: this difference arises from their having supposed the horizontal refraction greater than it appears to be from observation. In the dissertation of the last author, he proves, that above 7° altitude, it matters not whether you assume the refraction, according as it would be found from the increased density of the air, at the low altitudes, and which would give the refraction at 90° about 52′, or by an uniform density, which agrees better with observation; since, in the former case, it would only affect the refraction about two seconds.

Having thus assumed a greater value of y, than 3, for the coefficient of r, the mean refraction at 45° will vary inversely, as tang. $\overline{z - 3r}$: $58'',10734$: : tang. $\overline{z - yr} : x$. therefore,

$$x = \frac{\text{tang.}\ \overline{45^\circ - 3r} \times 58,10734}{\text{tang.}\ 45^\circ - yr} = 58'',1192.$$

The table of mean refractions by the proposed formula, compared with the observed refraction, differs with the lowest star, less than 2″; but the refractions at those low altitudes having always been found uncertain, we may presume that the result so nearly approximates the truth, that we may not possibly ascertain the refractions more correctly. However, since my deduction will soon be submitted to the test of some excellent circular instruments, it will be seen, whether more accurate tables can be formed.

M. BIOT, in the report of the National Institute, observes, that a third correction of the refractions having been supposed to arise from the vapours at low altitudes, and therefore an equation might be found from the hygrometer, he was induced to make some experiments with a prism containing warm air dried by potash, the outside of which was charged with the natural moisture of the atmosphere; but he could discover no alteration in the refraction. He consequently infers, that the hygrometrical state of the atmosphere has no sensible effect on the refractions. Supposing an effect really to exist, a standard hygrometer would be required; since those instruments, which show the comparative state of humidity or siccity of the atmosphere, give no scale of the velocity of the current of air which causes the change.

The mean astronomical refraction being determined in this manner, it will be further necessary to investigate the equations for the barometer and thermometer; and I must premise, that I have always applied those for the latter, from the temperature shewn by the thermometer attached to the telescope. Some astronomers have applied the correction for the external temperature, others for the mean of both; but I believe the majority have adopted the temperature within the observatory.

It is to be regretted, that astronomers have not informed us, by what methods they have deduced these corrections of the mean refractions; I shall therefore explain that which I have adopted, in order that this hypothesis may be investigated; and any error therein, be the more easily detected. I have taken all those stars in the table, below the Pole, from β Ursæ majoris to ε Aurigæ, both inclusive, having noticed that those observations made in winter, when the correction is additive, the assumed correction by Dr. BRADLEY's formula appears to be too great; and this difference will be seen by the comparison in Tables II. and III. To these I have added Fomalhaut, containing in the whole 210 observations. Having determined the mean place of the star at a certain epoch, I find the apparent place; the difference between which and the observed place, is the apparent refraction, which call a; put $b =$ mean refraction, $c =$ to the correction for the barometer, and $w =$ to the correction for the thermometer: then $\frac{a-b}{b} - c = w$.

The following observation of α Cygni will suffice for an example. Observed zenith distance 83° 47′ 58″,34; apparent zenith distance 83° 56′ 27″,12; apparent refraction 8′ 28″,78; mean refraction 8′ 7″,83; barometer 30,13; correction by BRADLEY + ,0179; thermometer within 41, without 34: then

$$\frac{a\ (8'.\ 28'',78) - b\ (8'.\ 7'',83)}{b\ (8'.\ 7'',83)} = +,0430 - c\,(+,0179) = w\,(+,0251)$$

for the thermometer within at 41, or without at 34.

Proceeding in this manner with the above 210 observations, I obtained equations for the thermometer within, from 31 to

79; and for the thermometer without, from 24 to 80. The difference of these extremes being divided by 8, will show the corrections for the former to every 6°, and for the latter to every 7°; and assuming that the variations are in a reciprocal ratio to the degrees of the thermometer, I propose the following formula. Putting h for the degree of FAHRENHEIT'S thermometer, then $\overline{49^\circ - h^\circ} \times ,0024$, when below the mean; $\overline{49^\circ - h^\circ} \times ,0023$, when above the mean; will be the correction for the thermometer within; and $\overline{45^\circ - h^\circ} \times ,0021$, the correction for the thermometer without. The Tables II. and III. show the near agreement of the observed and the equated factor, found as above. The barometer at its mean state may continue to be taken at 29,6 inches; and the mean point of the thermometer without, will then be at 45°. The thermometer within, which has been assumed at 50° for the mean state, from observation, appears to be at 49°; and supposing the barometer and thermometer to vary reciprocally, as the former in inches and the latter in degrees, the mean point of each is in an inverse ratio to the other, and might be changed if thought expedient. Indeed it appears from the mean of more than 1000 observations of these circumpolar stars, at all seasons, that the mean state of the barometer is 29,85, and of the thermometer 52°; yet, as the sum of the two corrections would still be the same, there cannot be occasion to alter the mean point of the former, especially as this might not be a general rule for every climate. It has very rarely happened, that I could make any observation, when the barometer has been below 29 inches, which causes the mean state to be so high as 29,85; but it may be otherwise in places, where the atmosphere is not so charged with vapours, as that of this country.

I have, in the Table IV. shewn the mean refractions, according to various authors; the first column contains those of Mr. SIMPSON, from the observations of Dr. BEVIS; the second, those of Dr. BRADLEY, by his formula, tang. $\overline{z - 3r} \times 57''$; but this quantity having been found, from later observations, to have been too small; the third column contains the same formula, into 58'',107; yet the refractions, at low altitudes, will be found too great, and this excess has been corrected by M. PIAZZI, using arbitrary equations, which will bring them nearer to the truth, as shown in the fourth column. The more uniform correction of the whole will be, as I have proposed in the above theorem, by increasing the coefficient of *r*; a corollary from which will be drawn, that the increment of the tangents near the horizon will cause the equations of M. PIAZZI to vanish. The fifth column contains the refractions of the French tables; and the sixth, my own; viz. tang. $\overline{z - 3{,}3625r} \times 58'',1192$.

On inspection of this table, it will appear, that the refractions by different formula do not vary considerably, so far as 80°; and thence to 87°, my deductions are rather in excess of the French tables; below which, mine are rather less. Whether this arises from the defect of the formula, I cannot determine; since the wall of Greenwich park being above my horizon, I cannot observe lower than 88½°: however, so far as 88°, the formula I have proposed, agrees with observation.

S. GROOMBRIDGE.

Blackheath, 22 Jan. 1810.

Factors for the correction of the Refraction, the mean result of 210 observations.

Thermometer within.

	Bradley.	Observed.	Equated.
31	+,0499	+,0437	+,0436
37	,0336	,0329	,0295
43	,0178	,0155	,0155
49	,0025	,0000	,0014
55	—,0125	—,0132	—,0126
61	,0275	,0286	,0267
67	,0425	,0444	,0407
73	,0575	,0512	,0548
79	,0725	,0688	,0688

Thermometer without.

	Observed.	Equated.
24	+,0436	+,0436
31	,0319	,0289
38	,0153	,0142
45	,0007	—,0005
52	—,0161	,0152
59	,0291	,0299
66	,0420	,0446
73	,0587	,0593
80	,0741	,0740

Factors for the correction of the Refraction, equated from the mean of the above, according to the proposed formula.

Table II.

Thermometer within.

28	+,0504
32	,0408
36	,0312
40	,0216
44	,0120
48	,0024
49	,0000
52	—,0069
56	,0161
60	,0253
64	,0345
68	,0437
72	,0529
76	,0621
80	,0713

Table III.

Thermometer without.

20	+,0525
25	,0420
30	,0315
35	,0210
40	,0105
45	,0000
50	—,0105
55	,0210
60	,0315
65	,0420
70	,0525
75	,0630
80	,0735
85	,0840

Table IV. The mean astronomical Refractions, according to several Authors.

Zenith dis.	Simpson.		Bradley. tang. $\overline{z - 3r}$ × 57″		Bradley. tang. $\overline{z - 3r}$ × 58,″107.		Piazzi		French Tables.		S. G.	
°	′	″	′	″	′	″	′	″	′	″	′	″
10	0	9	0	10,00	0	10,19	0	10,2	0	10,3	0	10,24
20	0	19	0	20,70	0	21,10	0	20,8	0	21,2	0	21,13
30	0	30	0	32,90	0	33,54	0	33,2	0	33,4	0	33,51
40	0	44	0	47,80	0	48,73	0	48,1	0	48,9	0	48,69
45	0	52	0	56,90	0	58,00	0	57,2	0	58,2	0	58,01
50	1	2	1	7,80	1	9,11	1	8,2	1	9,3	1	9,11
55	1	14,5	1	21,20	1	22,77	1	22,4	1	23,1	1	22,77
60	1	30	1	38,40	1	40,31	1	39,8	1	40,6	1	40,29
65	1	52	2	1,70	2	4,02	2	3,5	2	4,3	2	3,98
70	2	23	2	35,50	2	38,50	2	37,8	2	38,8	2	38,41
72	2	40	2	53,90	2	57,28	2	56,5	2	57,6	2	57,13
74	3	1	3	16,70	3	20,44	3	18,3	3	20,6	3	20,22
76	3	27	3	45,50	3	49,79	3	47,3	3	49,8	3	49,44
78	4	2	4	23,18	4	28,23	4	24,3	4	27,9	4	27,68
80	4	50	5	14,83	5	20,78	5	16,1	5	19,8	5	19,85
81	5	21	5	48,45	5	55,00	5	47,4	5	53,5	5	53,74
82	5	59	6	29,55	6	36,85	6	28,3	6	34,4	6	35,06
83	6	48	7	20,84	7	28,98	7	19,5	7	24,7	7	26,46
84	7	49	8	26,41	8	35,61	8	24,9	8	29,9	8	31,85
85	9	10	9	52,50	10	3,18	9	45,4	9	54,3	9	57,27
86	11	5	11	49,77	12	1,95	11	42,6	11	48,3	11	52,21
87	13	44	14	34,61	14	48,78	14	25,1	14	28,1	14	31,75
88	17	43	18	34,30	18	50,59	18	2,7	18	22,2	18	19,19
89	23	50	24	28,14	24	46,42	23	46,1	24	21,2	23	46,77
90	33	0	32	59,43	33	18,52	32	3,0	33	46,3	31	27,87

TABLE I. *Circumpolar Stars for the Refraction, reduced to the mean Zenith Distance on the First of January*, 1807.

Star.	No. Observ.	Corrected Zenith distance.	Comp. Refraction.	Sum or difference.	Mean Zenith distance.	Sum or difference.	Cor. of Refraction.	Sum or diff. of true Zen. dis.	Co-Latitude.
		° ′ ″	′ ″	′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
Ursæ min. Bode 4.	6 7	37 35 10,59 39 27 12,45	0 44,06 0 44,97	1 29,63	37 35 55,25 39 27 57,42	77 3 52,67	2,55	77 3 55,22	38 31 57,61
Cephei. Hev. 24.	16 8	37 14 4,10 39 48 16,32	0 42,93 0 48,95	1 31,88	37 14 47,03 39 49 5,27	77 3 52,30	2,61	77 3 54,91	38 31 57,45
Polaris.	35 41	36 47 55,60 40 14 28,52	0 42,49 0 47,18	1 29,67	36 48 38,09 40 15 15,70	77 3 53,79	2,56	77 3 56,35	38 31 58,17
24 Ursæ minor.	20 7	35 27 34,06 41 34 48,52	0 39,19 0 51,56	1 30,75	35 28 13,25 41 35 40,08	77 3 53,33	2,58	77 3 55,91	38 31 57,95
δ Ursæ minor.	27 5	35 5 21,71 41 57 0,22	0 38,78 0 52,42	1 31,20	35 6 0,49 41 57 52,64	77 3 53,13	2,59	77 3 55,72	38 31 57,86
Cameiop. Hev. 29.	6 9	33 44 25,40 43 17 56,38	0 38,12 0 53,61	1 31,73	33 45 3,52 43 18 49,99	77 3 53,51	2,61	77 3 56,12	38 31 58,06
ν Cephei.	5 7	33 44 11,56 43 18 11,70	0 38,33 0 52,30	1 30,63	33 44 49,89 43 19 4,00	77 3 53,89	2,57	77 3 56,46	38 31 58,23
Cameiop. Hev. 30.	7 9	32 3 5,85 44 59 13,51	0 36,26 0 57,01	1 33,27	32 3 42,11 45 0 10,52	77 3 52,63	2,65	77 3 55,28	38 31 57,64
ι Ursæ minor.	14 10	30 51 25,00 46 10 54,73	0 33,00 1 0,48	1 33,48	30 51 58,00 46 11 55,21	77 3 53,21	2,67	77 3 55,88	38 31 57,94
5 Ursæ minor.	8 5	25 4 45,80 51 57 27,75	0 26,00 1 13,80	1 39,80	25 5 11,80 51 58 41,55	77 3 53,35	2,84	77 3 56,19	38 31 58,09
β Ursæ minor.	14 8	23 28 12,07 53 34 0,30	0 23,94 1 17,03	1 40,97	23 28 36,01 53 35 17,33	77 3 53,34	2,87	77 3 56,21	38 31 58,10
γ 1 Ursæ minor.	5 8	21 3 2,10 55 59 4,63	0 21,35 1 24,18	1 45,53	21 3 23,45 56 0 28,81	77 3 52,26	3,00	77 3 55,26	38 31 57,63
γ 2 Ursæ minor.	5 9	21 2 48,68 55 59 18,44	0 21,33 1 24,32	1 45,65	21 3 10,01 56 0 42,75	77 3 52,77	3,01	77 3 55,78	38 31 57,89
ο Cephei.	30 5	15 35 5,97 61 26 48,13	0 15,70 1 42,87	1 58,57	15 35 21,67 61 28 31,00	77 3 52,67	3,37	77 3 56,04	38 31 58,02
α Dracon.	13 6	13 49 48,80 63 11 53,24	0 13,77 1 55,84	2 9,61	13 50 2,57 63 13 49,08	77 3 51,65	3,69	77 3 55,34	38 31 57,67
α Ursæ major.	13 4	11 19 11,17 65 42 25,14	0 11,15 2 4,36	2 15,51	11 19 22,32 65 44 29,50	77 3 51,82	3,86	77 3 55,68	38 31 57,84
η Dracon.	26 8	10 29 0,80 66 32 30,11	0 10,27 2 11,18	2 21,45	10 29 11,07 66 34 41,29	77 3 52,36	4,02	77 3 56,38	38 31 58,19
θ Dracon.	33 6	7 36 53,00 69 24 21,09	0 7,38 2 29,85	2 37,23	7 37 0,38 69 26 50,94	77 3 51,32	4,48	77 3 55,80	38 31 57,90
2 Lyncis.	9 7	7 35 31,82 69 25 45,39	0 7,84 2 25,31	2 33,15	7 35 39,66 69 28 10,70	77 3 50,36	4,36	77 3 54,72	38 31 57,36
δ Ursæ major.	14 5	6 38 12,07 70 22 53,50	0 6,56 2 39,16	2 45,72	6 38 18,63 70 25 32,66	77 3 51,29	4,71	77 3 56,00	38 31 58,00
β Cassiop.	22 9	6 36 56,92 70 24 13,02	0 6,62 2 35,33	2 41,95	6 37 3,54 70 26 48,35	77 3 51,89	4,61	77 3 56,50	38 31 58,25
β Ursæ major.	29 6	5 56 42,74 71 4 18,90	0 5,84 2 43,84	2 49,68	5 56 48,58 71 7 2,74	77 3 51,32	4,83	77 3 56,15	38 31 58,07
ι Ursæ major.	26 7	5 32 27,53 71 28 28,95	0 5,42 2 49,39	2 54,81	5 32 32,95 71 31 18,34	77 3 51,29	4,97	77 3 56,26	38 31 58,13
ζ Ursæ major.	31 7	4 28 5,41 72 32 37,94	0 4,35 3 2,96	3 7,31	4 28 9,76 72 35 40,90	77 3 50,56	5,33	77 3 55,99	38 31 57,99
η Persei.	10 9	3 36 58,63 73 23 44,29	0 3,65 3 4,10	3 7,75	3 37 2,28 73 26 48,39	77 3 50,67	5,34	77 3 56,01	38 31 58,00
γ Ursæ major.	20 9	3 17 59,02 73 42 35,84	0 3,23 3 12,08	3 15,31	3 18 2,25 73 45 47,92	77 3 50,17	5,56	77 3 55,73	38 31 57,86
δ Aurigæ.	9 6	2 47 3,31 74 13 31,93	0 2,84 3 10,28	3 13,12	2 47 6,15 74 16 42,21	77 3 48,36	5,49	77 3 53,85	38 31 56,92
θ Cassiop.	10 7	2 39 6,33 74 21 22,98	0 2,69 3 16,87	3 19,56	2 39 9,02 74 24 39,85	77 3 48,87	5,68	77 3 54,55	38 31 57,27
θ Bootis.	30 4	1 16 47,07 75 43 16,44	0 1,24 3 45,70	3 46,94	1 16 48,31 75 47 2,14	77 3 50,45	6,46	77 3 56,91	38 31 58,45
γ Persei.	7 9	1 16 20,81 75 43 51,84	0 1,25 3 35,73	3 36,98	1 16 22,06 75 47 27,57	77 3 49,63	6,17	77 3 55,80	38 31 57,90
ξ Ursæ major.	14 9	1 4 52,51 75 55 13,18	0 1,08 3 42,08	3 43,16	1 4 53,59 75 58 55,26	77 3 48,85	6,35	77 3 55,20	38 31 57,60
β Dracon.	33 7	0 58 54,36 76 1 2,45	0 0,96 3 51,31	3 52,27	0 58 55,32 76 4 53,76	77 3 49,08	6,61	77 3 55,69	38 31 57,84
γ Dracon.	58 8	0 2 57,37 76 56 44,22	0 0,05 4 7,90	4 7,95	0 2 57,42 77 0 52,12	77 3 49,54	7,05	77 3 56,59	38 31 58,29
η Ursæ major.	57 10	1 11 11,67 78 10 30,62	0 1,16 4 30,98	4 29,82	1 11 12,83 78 15 1,60	77 3 48,77	7,94	77 3 56,71	38 31 58,35
α Persei.	20 12	2 18 14,89 79 17 19,00	0 2,30 4 46,90	4 44,60	2 18 17,19 79 22 5,90	77 3 48,71	8,09	77 3 56,80	38 31 58,40
θ Persei.	15 8	3 3 48,89 80 2 32,59	0 3,11 5 7,69	5 4,58	3 3 52,00 80 7 40,28	77 3 48,28	8,67	77 3 56,95	38 31 58,47
τ Herculis.	5 5	4 41 14,02 81 38 50,33	0 4,47 6 15,64	6 11,17	4 41 18,49 81 45 5,97	77 3 47,48	10,56	77 3 58,04	38 31 59,02
Capella.	24 19	5 40 45,74 82 37 51,21	0 5,63 6 47,87	6 42,24	5 40 51,37 82 44 39,08	77 3 47,71	11,44	77 3 59,15	38 31 59,57
ψ Ursæ major.	14 5	5 55 19,56 82 52 3,33	0 5,85 7 9,74	7 3,89	5 55 25,41 82 59 13,07	77 3 47,66	12,06	77 3 59,72	38 31 59,86
β Aurigæ.	9 11	6 33 13,88 83 29 32,80	0 6,50 7 34,35	7 27,85	6 33 20,38 83 37 7,15	77 3 46,77	12,74	77 3 59,51	38 31 59,75
α Cygni.	26 6	6 52 9,82 83 47 39,78	0 6,82 8 23,42	8 16,60	6 52 16,64 83 56 3,20	77 3 46,56	14,13	77 4 0,69	38 32 0,34
λ Ursæ major.	9 5	7 35 31,84 84 30 21,78	0 7,62 9 6,55	8 58,93	7 35 39,46 84 39 28,33	77 3 48,87	15,33	77 4 4,20	38 32 2,10
ι Aurigæ.	27 8	7 56 31,11 84 51 8,35	0 8,07 9 17,51	9 9,44	7 56 39,18 85 0 25,86	77 3 46,68	15,63	77 4 2,31	38 32 1,15
10 Ursæ major.	6 5	8 55 37,75 85 48 26,95	0 9,07 11 17,66	11 8,59	8 55 46,82 85 59 44,61	77 3 57,79	19,02	77 4 16,81	38 32 8,40
μ Ursæ major.	14 5	8 59 58,08 85 52 23,49	0 9,01 11 39,45	11 30,44	9 0 7,09 86 4 2,94	77 3 55,85	19,64	77 4 15,49	38 32 7,74
γ Androm.	12 4	10 4 1,34 86 54 21,52	0 10,20 13 51,03	13 40,83	10 4 11,54 87 8 12,55	77 4 1,01	23,35	77 4 24,36	38 32 12,18
β Bootis.	8 2	10 18 23,26 87 7 35,00	0 10,02 15 0,17	14 50,15	10 18 33,28 87 22 35,17	77 4 1,89	25,33	77 4 27,22	38 32 13,61
η Aurigæ.	17 4	10 30 17,75 87 19 18,98	0 10,78 15 3,00	14 52,22	10 30 28,53 87 34 21,98	77 3 53,45	25,38	77 4 18,83	38 32 9,41
ζ Aurigæ.	9 2	10 41 4,42 87 29 7,32	0 10,91 15 58,10	15 47,19	10 41 15,33 87 45 5,42	77 3 50,09	26,94	77 4 17,03	38 32 8,51
β Persei.	11 6	11 15 42,06 88 1 23,30	0 11,29 18 24,73	18 13,44	11 15 53,35 88 19 48,03	77 3 54,68	31,11	77 4 25,79	38 32 12,89